**Portfolio Active Return Decomposition**

1. We start with decomposing portfolio active weight vector

Where:

* is the active weight of stock i
* is the weight of alpha weighted portfolio, rescaled to be 100% long + 100% short
* is the active weight resulted from constraint, it is unknown and will be calculated

1. We proceed with decomposing :

where:

* scalar is a constant rescaling alpha to long-short weight, it is the same for any standardized score
* is the weight of factor j when forming the alpha
* is the factor score vector of factor j

So we have:

where , and is the weight of factor score weighted portfolio.

1. From 1) and 2) we have following active weight decomposition:

Multiply the equation by the stock return vector **R:**

We now have the active return decomposition.

We further define:

The math is clean and no residual left. However, when the final alpha is normalized again, the formula in 2) won’t hold exactly, then the needs to be found through regression, which will lead to a small attribution error term.